What is "in" Semidefinite Programming?

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ORSUM

2004

? what ?

? why ?

? when ?

? how ?

SemiDefinite Programming (SDP)

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. . . generalization of Linear Programming

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… applications: global and comb. optim., in engineering, …

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- $\langle Y, X \rangle = \mathrm{tr}(Y^T X) \ldots$ trace inner product

Semidefinite Programming

$$(\text{PSDP}) \qquad \textbf{min} \ \langle C, X \rangle$$
$$\text{s. t.} \quad \mathcal{A}X = a$$
$$X \succeq 0$$

where

- $C, X \in \mathcal{S}_n, \quad a \in I\!R^m$
- $\mathcal{A} : \mathcal{S}_n \to I\!R^m$ linear operator

Linear Transformation $\mathcal{A}$

$\mathcal{A} : \mathcal{S}_n \to \mathbb{R}^m$

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$$\mathcal{A}X = \begin{pmatrix} \langle A_1, X \rangle \\ \vdots \\ \langle A_m, X \rangle \end{pmatrix}$$

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$\mathcal{A}^* : \mathbb{R}^m \to \mathcal{S}_n$

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$$\Rightarrow \quad \mathcal{A}^* w = \sum_{i=1}^{m} w_i A_i$$

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- PSDP $\Leftrightarrow \min_{X \succeq 0} \max_{w} \mathcal{L}(X, w)$

- minimax inequality:

$$\min_{X \succeq 0} \max_{w} \mathcal{L}(X, w) \geq \max_{w} \min_{X \succeq 0} \mathcal{L}(X, w)$$

- rewrite $\mathcal{L}$:

$$\mathcal{L}(X, w) = \langle a, w \rangle + \langle C - \mathcal{A}^*(w), X \rangle$$

- note

$$\min_{X \succeq 0} \mathcal{L}(X, w) = \begin{cases} \langle a, w \rangle & \text{if } C - \mathcal{A}^*(w) \succeq 0 \\ -\infty & \text{otherwise} \end{cases}$$

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- dual of PSDP:

$$\begin{array}{lll} \text{(DSDP)} & \textbf{max} & \langle a, w\rangle \\ & \text{s. t.} & C - \mathcal{A}^*(w) = Z \\ & & Z \succeq 0 \end{array}$$

Duality Theory

Duality Theory

$X \succeq 0$ feasible (PSDP) $\leadsto \mathcal{A}X = a$

$(w, Z),\ Z \succeq 0$ feasible (DSDP) $\leadsto L = \mathcal{A}^T w + Z$

Duality Theory

duality gap:

$$\langle C, X\rangle - \langle a, w\rangle = \langle Z + \mathcal{A}^*(w), X\rangle - \langle \mathcal{A}X, w\rangle = \langle Z, X\rangle \geq 0$$

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weak duality:
if $X \succeq 0$, $\mathcal{A}X = a$ and $w \in \mathbb{R}^m$, $Z = C - \mathcal{A}^*(w) \succeq 0$, then

$$\langle C, X\rangle \geq \langle a, w\rangle$$

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strong duality
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optimality conditions:

$$\begin{aligned} \mathcal{A}X &= a && \text{primal feasibility} \\ Z + \mathcal{A}^*(w) &= C && \text{dual feasibility} \\ ZX &= 0 && \text{complementarity slackness} \end{aligned}$$

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- provide the basis for:
  interior point methods,
  primal simplex method, dual simplex method

Examples

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$$\min_{y} \; \lambda_{\max}(A(y))$$

$$A(y) = A_0 + y_1 A_1 + \ldots + y_m A_m,$$

for given $A_i \in \mathcal{S}_n$, $i = 0, \ldots, m$

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$\lambda_{\max}(A(y)) \leq \alpha$

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$$\min\{\alpha : \alpha I - A(y) \succeq 0\}$$

Quadratic Optimization Problem

$$\begin{array}{ll} \min & f_0(x) \\ \text{s.t.} & f_i(x) \leq 0, \quad i = 1, \ldots, m \end{array}$$

where

$$f_i(x) = x^T A_i x + 2 b_i^T x + c_i, \quad i = 0, 1, \ldots, m$$

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$$\text{s.t.} \quad \begin{pmatrix} A_0 & b_0 \\ b_0^T & c_0 - t \end{pmatrix} + \tau_1 \begin{pmatrix} A_1 & b_1 \\ b_1^T & c_1 \end{pmatrix} + \ldots + \tau_m \begin{pmatrix} A_m & b_m \\ b_m^T & c_m \end{pmatrix} \succeq 0$$

$$\tau_i \geq 0, \quad i = 1, \ldots, m$$

Further Application Areas

- generalized eigenvalue problems
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and

Combinatorial Optimization

- **Max–Cut Problem**
- **k–Equipartition Problem**
- **Quadratic Assignment Problem**
- **Quadratic Knapsack Problem**
- . . . .

Max–Cut

…on undir. edge–weighted graph with vert. set $V := \{1, \ldots, n\}$

PROBLEM: Find partition V into S and $V \setminus S$ s.t. the total weight of the edges joining S and $V \setminus S$ is maximized

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$$z_{mc} = \max \{x^T L x : x \in \{-1, 1\}^n\}$$

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NP–hard problem!

applications of MC:

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techniques used for general MC:

- heuristic
- integer programming (branch–and–bound)
- approximation algorithms

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Goemans & Williamson ('94) proved:

$z_{mc-basic}$ has error of $\leq 13.82\%$ (!)

? can it be improved

strengthened $z_{mc-basic}$ $\rightsquigarrow$ $Y \in$ MET := triangle inequalities

MET

$$\begin{aligned} y_{ij} + y_{ik} + y_{jk} &\geq -1 \\ y_{ij} - y_{ik} - y_{jk} &\geq -1 \\ -y_{ij} + y_{ik} - y_{jk} &\geq -1 \\ -y_{ij} - y_{ik} + y_{jk} &\geq -1, \ \forall i < j < k \end{aligned}$$

strengthened $z_{mc-basic}$ $\rightsquigarrow$ $Y \in$ MET := triangle inequalities

$$\text{MET} \quad \begin{array}{rcl} y_{ij} + y_{ik} + y_{jk} & \geq & -1 \\ y_{ij} - y_{ik} - y_{jk} & \geq & -1 \\ -y_{ij} + y_{ik} - y_{jk} & \geq & -1 \\ -y_{ij} - y_{ik} + y_{jk} & \geq & -1, \ \forall i < j < k \end{array}$$

$$z_{mc-met} = \max\{\operatorname{tr}(LY) : \operatorname{diag}(Y) = e, \ Y \succeq 0, Y \in \text{MET}\}$$

- z_{mc-met} is SDP with $n + 4 \binom{n}{3}$

Modelling 0–1 Variables

$B := \{x : x \in \{0,1\}^n\}$ … set of binary n vectors

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? the key idea

use: $Y \in \mathcal{S}_n$ where

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$$Y = xx^T$$

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! exploit 0–1 properties of x:

$$x_i x_j \geq 0, \;\; x_i(1 - x_j) \geq 0, \;\; (1 - x_i)(1 - x_j) \geq 0$$

$$y_{ij} \geq 0, \qquad y_{ii} \geq y_{ij}, \qquad 1 + y_{ij} \geq y_{ii} + y_{jj}$$

! further strengthening

- $Y \succeq 0$

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- $Y - \text{diag}(Y)\text{diag}(Y)^T \succeq 0$ ... nonlinear in Y

but

$\Leftrightarrow \mathcal{A}(Y) + E_{n+1} \succeq 0$ where

$$\mathcal{A}(Y) := \begin{pmatrix} Y & \text{diag}(Y) \\ \text{diag}(Y)^T & 0 \end{pmatrix} \text{ and } E_{n+1} = e_{n+1}^T e_{n+1}$$

- representing quad. constr.: $x^T D x \leq f$

  as lin. ineq. : $\mathrm{tr}(DY) \leq f$

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  diag. repr. : $\mathrm{tr}(\mathrm{Diag}(a_i)Y) \leq b_i$

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  OR

  squared repr. : $(a_i^T x)^2 \le b_i^2$

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  OR

  extended squared repr. : $(a_i^T x)^2 \leq b_i(a_i^T x)$

  that is: $\mathrm{tr}(a_i a_i^T - b_i \mathrm{Diag}(a_i), Y) \leq 0$

!! OR

!! OR

- multiply $a_i^T x - b_i \leq 0$ by x_j and $1 - x_j$

$$(a_i^T x)x_j - b_i x_j \leq 0, \quad (a_i^T x)(1 - x_j) - b_i(1 - x_j) \leq 0$$

define

$$M_{ij} := \frac{1}{2}(a_i e_j^T + e_j a_i^T) - b_i e_j e_j^T$$
$$N_{ij} := \mathrm{Diag}(a_i) - \frac{1}{2}(a_i e_j^T + e_j a_i^T) + b_i e_j e_j^T$$

now

$$\mathrm{tr}(M_{ij} Y) \leq 0, \quad \mathrm{tr}(N_{ij} Y) - b_i \leq 0$$

. . . and much, much more . . .

Algorithmic Approaches

- Interior Point Methods
- Spectral Bundle Method
- Bundle Methods
- Low Rank Factorization
- Solving Semidefinite Programs via Nonlinear Programming
- ....

Softwares

- SeDuMi
- SDPA, (SDPARA – parallel version)
- SDPT3
- SCDP
- PENNON
- ....

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all available on internet and FREE !!

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- ....
- solving optim. problems via internet: NEOS server
  check: http://www-neos.mes.anl.gov/

Books

- M. Laurent and F. Rendl.
  Semidefinite Programming and Integer Programming.
  A preliminary version: Report PNA-R0210, CWI, Amsterdam, 2002.
- E. de Klerk.
  Aspects of Semidefinite Programming: Interior Point Algorithms and Selected Applications.
  Applied Optimization Series, Volume 65. Kluwer, 2002.
- H. Wolkowicz, R. Saigal, and L. Vandenberghe (editors).
  Handbook on Semidefinite Programming. Kluwer, 2000.
- . . . .

?? TIRED

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?! You want more informations

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check Christoph Helmberg's SDP page
(with links to papers and software downloads)
http://www-user.tu-chemnitz.de/ helmberg/semidef.html

Thank You!

Problems Involving Moments

- if $x_1, \ldots, x_{2n}$ are *moments* of some distribution, then

$$H(x_0, \ldots, x_{2n}) = \begin{pmatrix} 1 & x_1 & \ldots & x_n \\ x_1 & x_2 & \ldots & x_{n+1} \\ \vdots & \vdots & \vdots & \vdots \\ x_n & x_{n+1} & \ldots & x_{2n} \end{pmatrix} \succeq 0$$

- find distribution with max variance and $l_i \leq x_i \leq u_i$

$$
\begin{aligned}
\max \quad & y \\
\text{s.t.} \quad & \begin{pmatrix} x_2 - y & x_1 \\ x_1 & 1 \end{pmatrix} \succeq 0 \\
& l_i \leq x_i \leq u_i, \quad i = 1, \ldots, 2m \\
& H(1, x_1, \ldots, x_{2n}) \succeq 0
\end{aligned}
$$